Orthogonal Delay-Doppler Division Multiplexing (ODDM)

A Novel Delay-Doppler Domain Multi-Carrier Waveform for NextG

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Outline

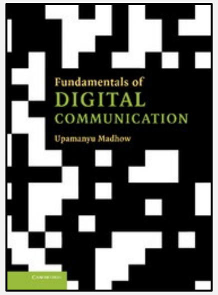
Introduction

"Modulation refers to the representation of digital information in terms of analog waveforms that can be transmitted over physical channels."

-- Upamanyu Madhow

Fundamental of Digital Communication

Cambridge University Press, 2008



Waveform

$$x(t) = X_i g_i(t), i \in \mathbb{Z}$$

analog pulse/filter

digital symbol

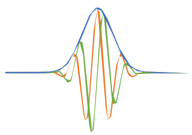
Function set

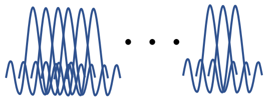
$$\{g_i(t), i \in \mathbb{Z}\}$$

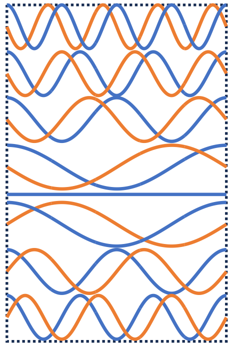
- A digital modulation waveform is an analog waveform assembled by digitally modulated pulses/filters/continuous-time functions

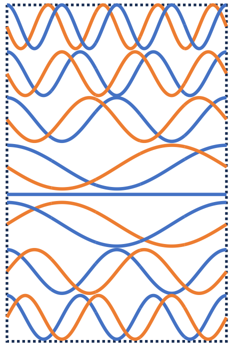
Introduction

Cellular Evolution	1G (1980's)	2G (1990's)	3G (2000's)	4G (2010's)	5G (2020's)
Data Rate	2.4 kbps	64 kbps	100 kbps - 56 Mbps	Up to 1 Gbps	> 1 Gbps
Carrier Frequency	800-900 MHz	850-1900MHz	1.6-2.5GHz	2-8 GHz	Sub- 6GHz, mmWave
Modulation	Analog FDMA	TDMA	CDMA	OFDM	OFDM
Pulse/Filter	N/A	Gaussian	RRC (chip) pulses modulated by spreading code	Complex-Sinusoids/ Subcarriers/Tones truncated by rectangular pulse	Complex-Sinusoids/ Subcarriers/Tones truncated by rectangular pulse





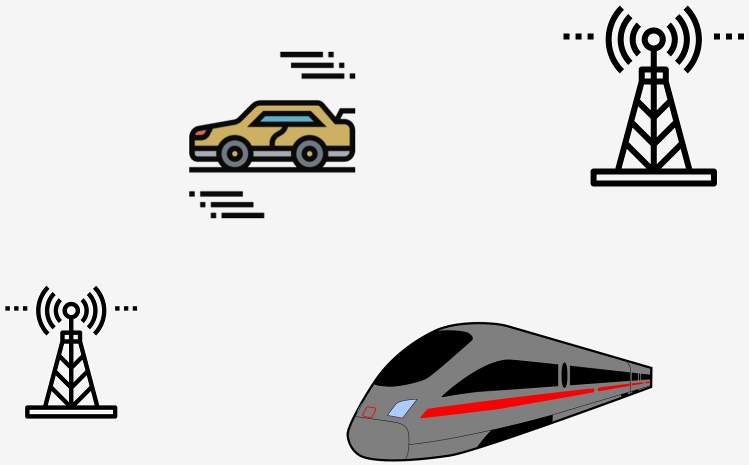




- Modulation waveforms are designed to support high rate and deal with fading and interference.

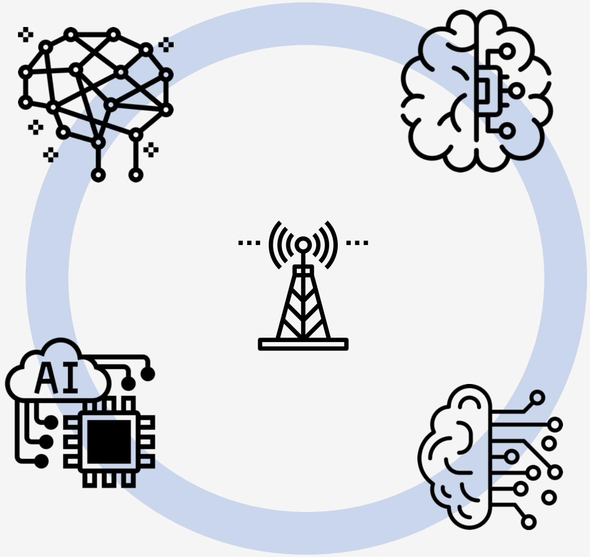
Potential NextG Scenarios

- High Mobility



High Reliability Communication (HRC)

- Connected Intelligence



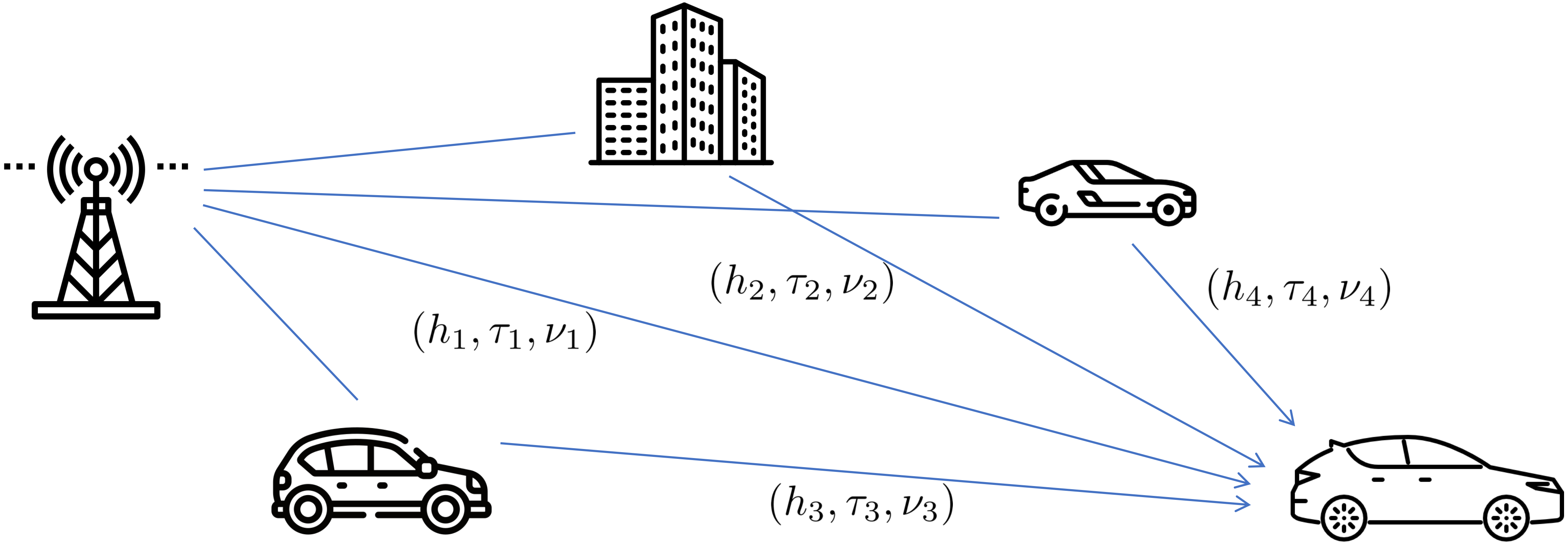
Integrated Sensing and Communication (ISAC)

Any signal waveform better interacting with doubly-selective wireless channels?

- Robust to distortion of doubly-selective channels: Path-diversity
- Viable for ISAC over doubly-selective channels: Fine TF resolutions

} an impulsive pulse in TF domain? ("narrow" in both time and frequency)

Mobile Channel Models



- Doubly-selective channel with both time and frequency dispersions
- Statistical models: WSSUS, Rayleigh, Rician, Nakagami-m
- Deterministic model: delay-Doppler spread function, namely spreading function $\mathcal{S}(\tau, \nu), h(\tau, \nu)$

DD Domain Modulation?

- Channel IO relation:

$$y(t) = \sum_{p=1}^{P} h_p x(t-\tau_p) e^{j2\pi\nu_p(t-\tau_p)}$$

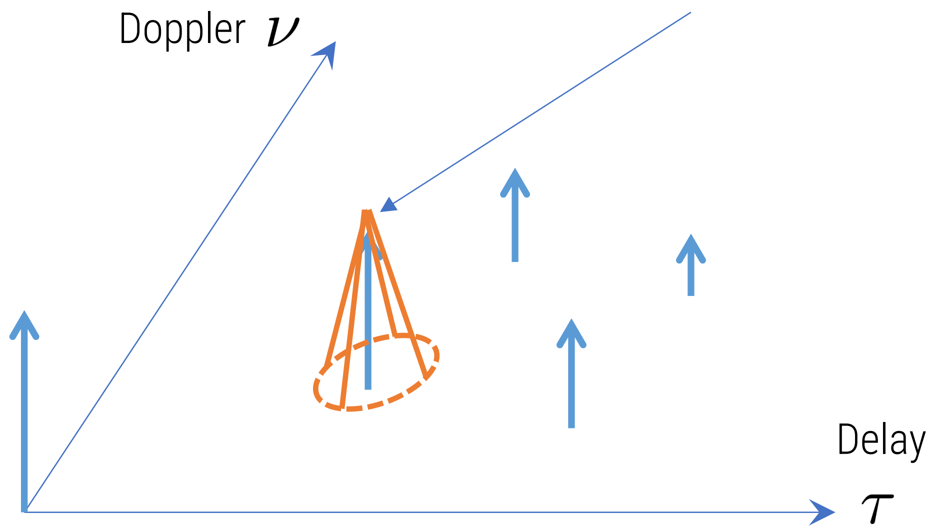
- Delay-time channel: Fading

$$h(\tau, t) = \sum_{p=1}^{P} h_p e^{j2\pi\nu_p t} \delta(\tau - \tau_p)$$

- Delay-Doppler channel: Diversity

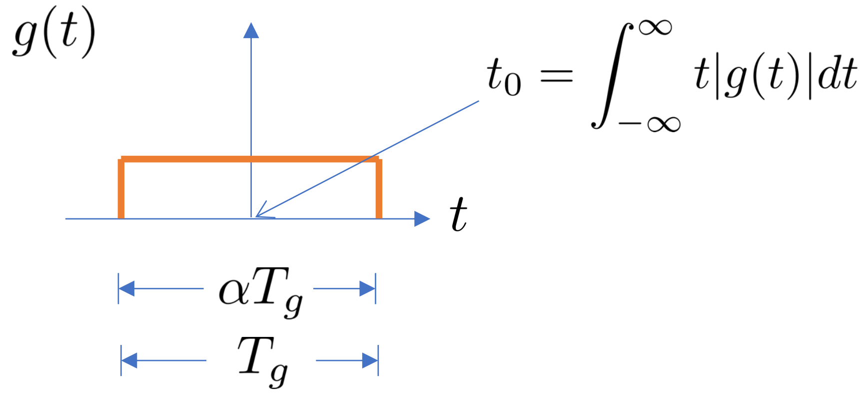
$$h(\tau, \nu) = \sum_{p=1}^{P} h_p \delta(\tau - \tau_p)\delta(\nu - \nu_p)$$

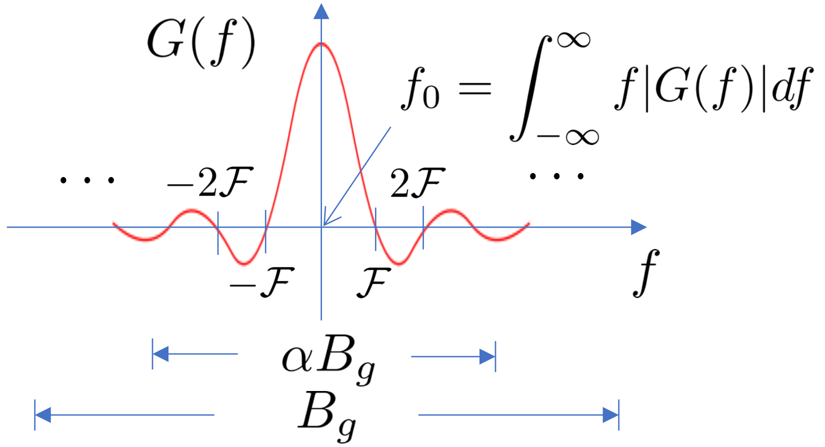
DDLP doesn't exist.
(violates the uncertainty principle!)



- DD domain modulation aims at path diversity, was first considered in the OTFS modulation.
- It seems that DD domain modulation requires an "impulse" or DD domain localized pulse (DDLP)
- However, a practical pulse cannot be "narrow" in time and frequency simultaneously.

Time-Frequency Area (TFA) of Pulse





Normalized $g(t)$ with $t_0 = 0, f_0 = 0$

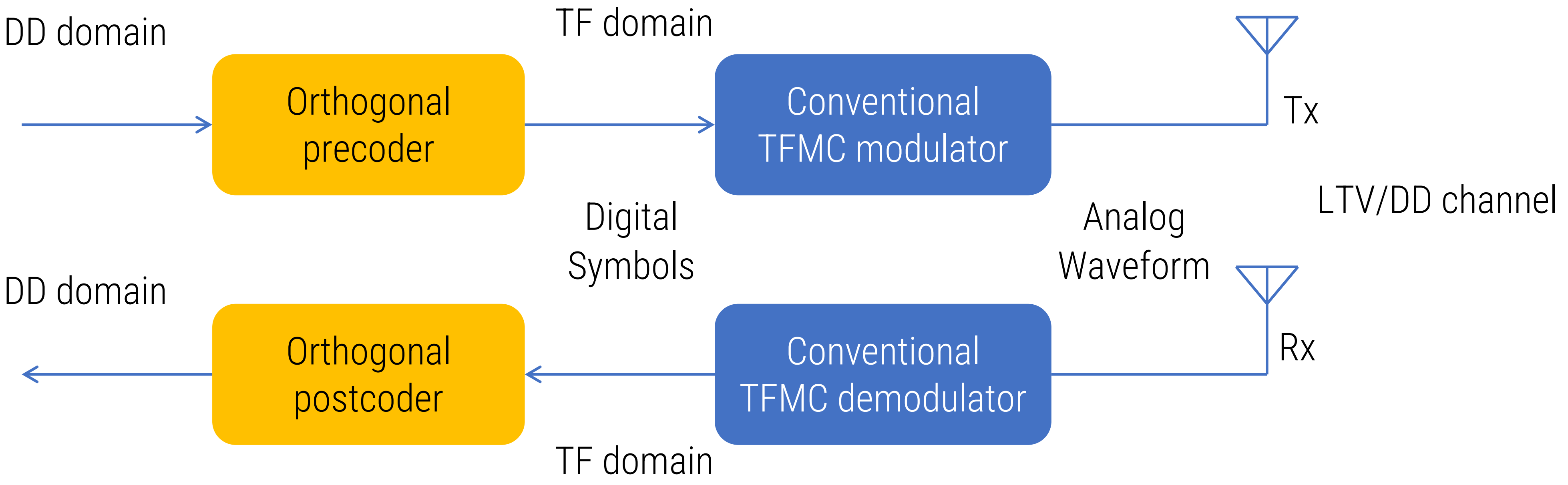
$$(\alpha T_g)^2 = \int_{-\infty}^{\infty} t^2|g(t)|^2 dt$$

$$(\alpha B_g)^2 = \int_{-\infty}^{\infty} f^2|G(f)|^2 dt$$

- TFA $= \alpha T_g \alpha B_g$ is a classic metric of pulse's TF occupancy (αT_g: time dispersion, αB_g: frequency dispersion)

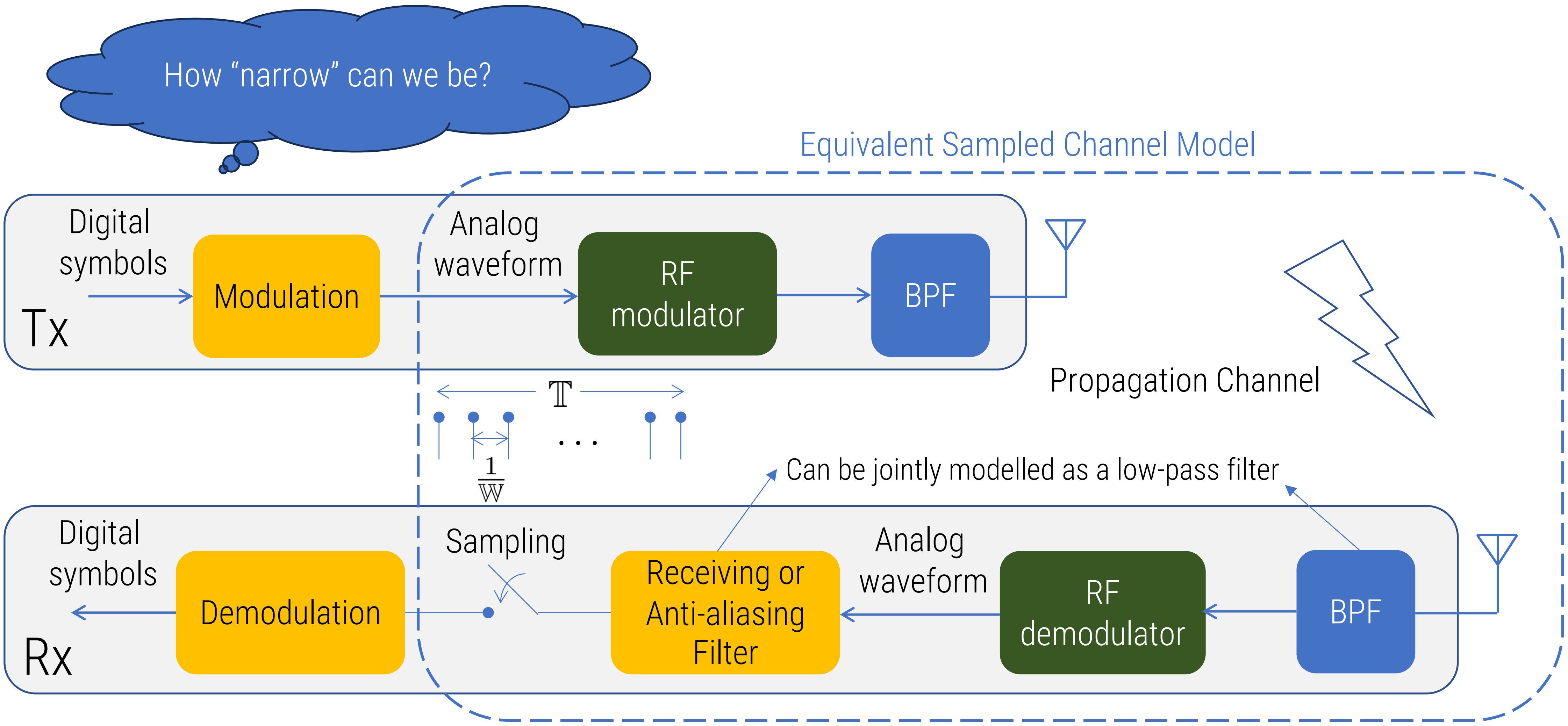
- According to the Uncertainty Principle**,** the TFA is bounded by the Gabor limit $\alpha T_g \alpha B_g \geq \frac{1}{4\pi} \approx 0.0796.$
- Gabor limit is achieved by Gaussian pulse, a pulse with a properly small TFA is said to be TF well-localized**.**

OTFS Modulation



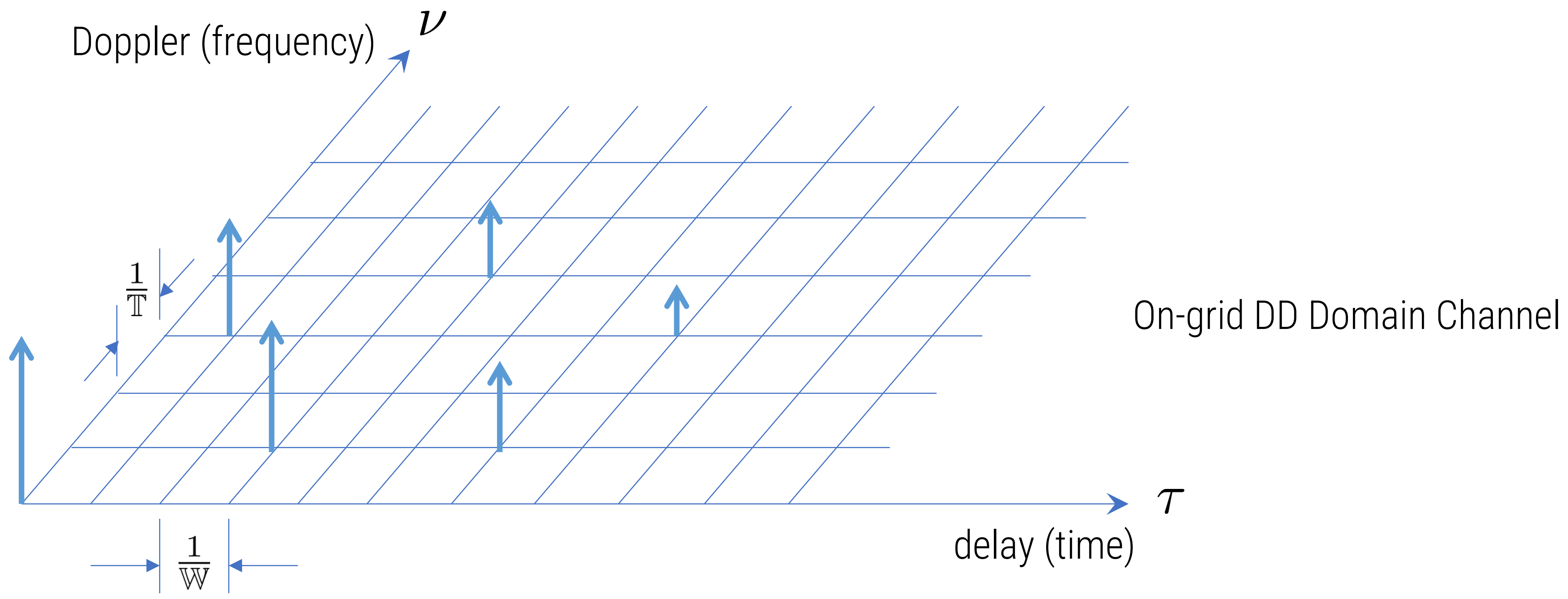
- **Issue/Consideration** : Due to lack of pulse, DD domain is considered as different from TF domain.
- **Approach**: Use orthogonal precoding (ISFFT or Walsh-Hadamard) to transfer data from DD to TF domain, then employ conventional multi-carrier (MC) modulator, such as OFDM and FBMC
- **Result**: OTFS relies on its employed TF domain MC modulation (TFMC) waveform

Reconsider DD domain



- P. Bello, "Characterization of randomly time-variant linear channels," IEEE Trans. Commun. Syst., vol. 11, no. 4, pp. 360–393, 1963.

Equivalent Sampled DD Channel



- We observe an on-grid DD domain in practice, due to the limited bandwidth and duration of signal.
- Physical unit of delay and Doppler are time and frequency, respectively.
- On-grid DD domain is exactly an on-grid TF domain : Frequency grid -> Multi-Carrier
- Without DDLP, can we design a DD domain multi-carrier (DDMC) modulation?

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Multi-Carrier (MC) Modulation

Such analog pulses are also known as Weyl Heisenberg (WH) /Gabor set

$$x(t) = \sum_{m=0}^{M-1} \sum_{n=-\frac{N}{2}}^{\frac{N}{2}-1} X[m,n] g(t - m\Delta T) e^{j2\pi n\Delta F(t-m\Delta T)}$$

- $X[m,n]$: digital symbol drawn from a constellation
- $g(t - m\Delta T)$: Prototype pulse usually orthogonal w.r.t $\Delta T, \Delta F$
- $e^{j2\pi n\Delta F(t-m\Delta T)}$: Subcarrier, a.k.a eigenfunction of LTI channel

➢ At the Rx, receive filtering (matched filtering or correlators) first then equalization

➢ Therefore, (bi)orthogonal pulses (if possible, even after channel distortion) are desired.

✓ G. Matz, H. Bolcskei, and F. Hlawatsch, "Time-frequency foundations of communications: Concepts and tools," IEEE Signal Process. Mag., vol. 30, no. 6, pp. 87–96, 2013.

✓ B. Le Floch, M. Alard and C. Berrou, "Coded orthogonal frequency division multiplex," Proc. IEEE, vol. 83, no. 6, pp. 982-996, 1995.

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(Bi)Orthogonal WH/Gabor Sets

- Fundamental tool of time-frequency analysis for signals/functions
- Gabor (Weyl-Heisenberg, Short-time/Windowed Fourier) expansion
- For signals lie in space $L^2(\mathbb{R})$

$$x(t) = \sum_{m\in\mathbb{Z}} \sum_{n\in\mathbb{Z}} c_{m,n} g_{m,n}(t), \quad g_{m,n}(t) = g(t - m\Delta T) e^{j2\pi n \Delta F (t - m\Delta T)}$$

$g(t)$: Gabor atom (function), prototype pulse

$$\hat{c}_{m,n} = \langle x, \gamma_{m,n} \rangle = \int x(t) \gamma^*_{m,n}(t) dt$$

$$\gamma_{m,n}(t) = \gamma(t - m\Delta T) e^{j2\pi n \Delta F (t - m\Delta T)}$$

- WH sets: $(g, \Delta T, \Delta F) = \{g_{m,n}(t)\}_{m,n\in\mathbb{Z}}, \ (\gamma, \Delta T, \Delta F) = \{\gamma_{m,n}(t)\}_{m,n\in\mathbb{Z}}$
- WH frames: Complete or overcomplete WH sets with guaranteed numerical stability of reconstruction

$\Delta R = \Delta T \Delta F$

JTFR	Sampling	Completeness	Frame for $(g, \frac{1}{\Delta F}, \frac{1}{\Delta T}), (\gamma, \frac{1}{\Delta F}, \frac{1}{\Delta T})$	(Bi)orthogonal WH sets exist?
$\Delta R > 1$	Under-Critical	Incomplete	✓ dual/tight	Yes
$\Delta R = 1$	Critical	Complete	✓ dual/tight	Yes
$\Delta R < 1$	Over-Critical	Overcomplete	× dual/tight	No

MC Modulation Parameters

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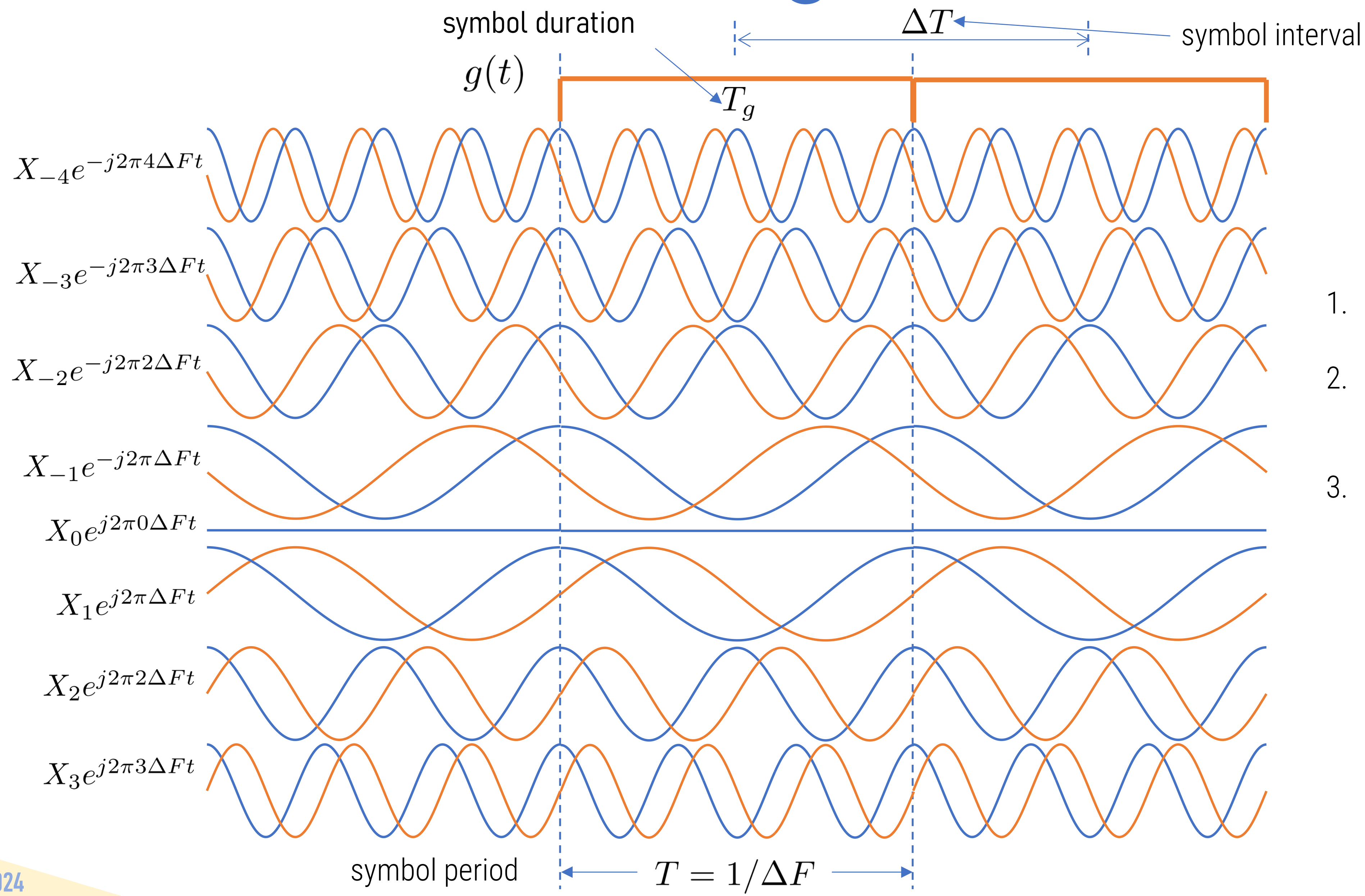
Notation	Parameters
ΔF	Frequency resolution, subcarrier spacing, fundamental frequency
T	Symbol period, $T = 1/\Delta F$
ΔT	Time resolution, symbol interval
ΔR	Joint time-frequency resolution, $\Delta R = \Delta T \Delta F$
N	Number of subcarriers
M	Number of symbols
$g(t)$	Transmit prototype pulse
T_g	Duration of $g(t)$, symbol duration
$G(f)$	Fourier transform of $g(t)$
B_g	Bandwidth of $g(t)$, span of $G(f)$
αT_g	Time dispersion of $g(t)$, standard deviation of $g(t)$
αB_g	Frequency dispersion of $g(t)$, standard deviation of $G(f)$

Core of modulation design, traditionally bounded by the WH frame theory

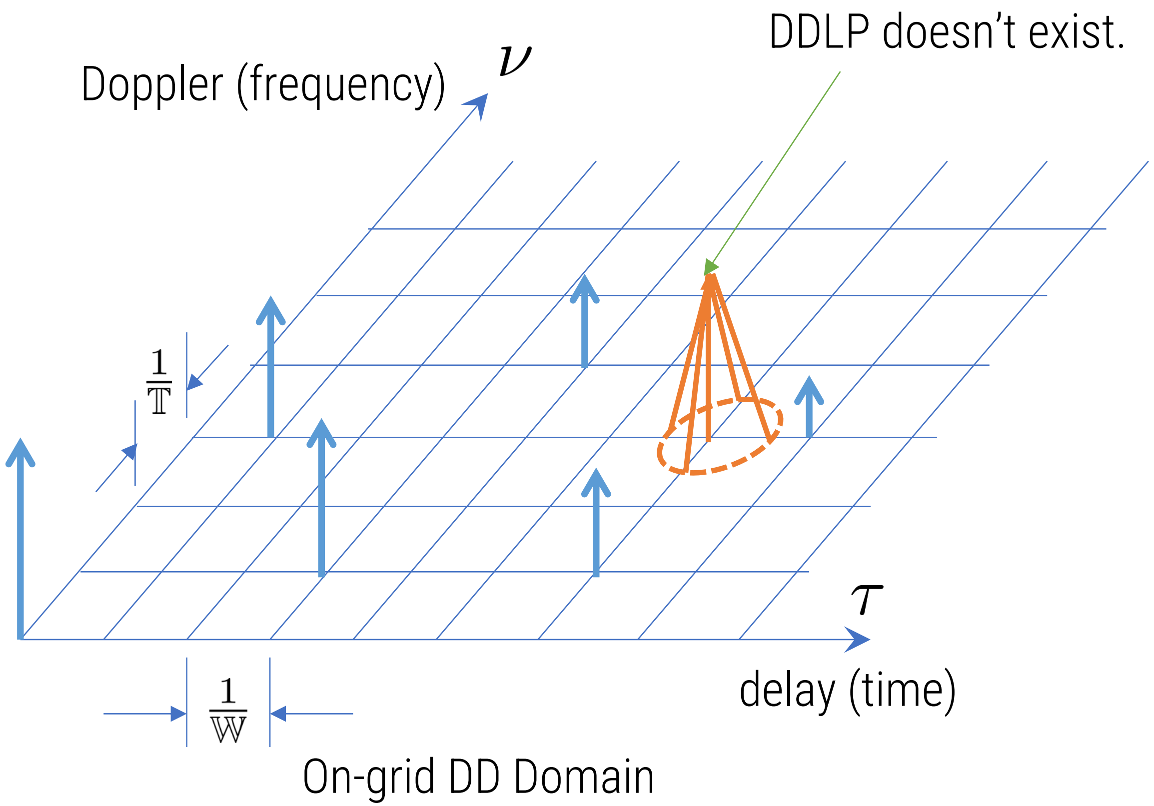
Bounded by the uncertainty principle

MC/OFDM: Truncating the Subcarriers



1. Given ΔF and $T = 1/\Delta F$
2. For ΔT, find $g(t)$ (pulse-shaped OFDM)
3. Truncate the subcarriers using $g(t)$

DDMC Modulation



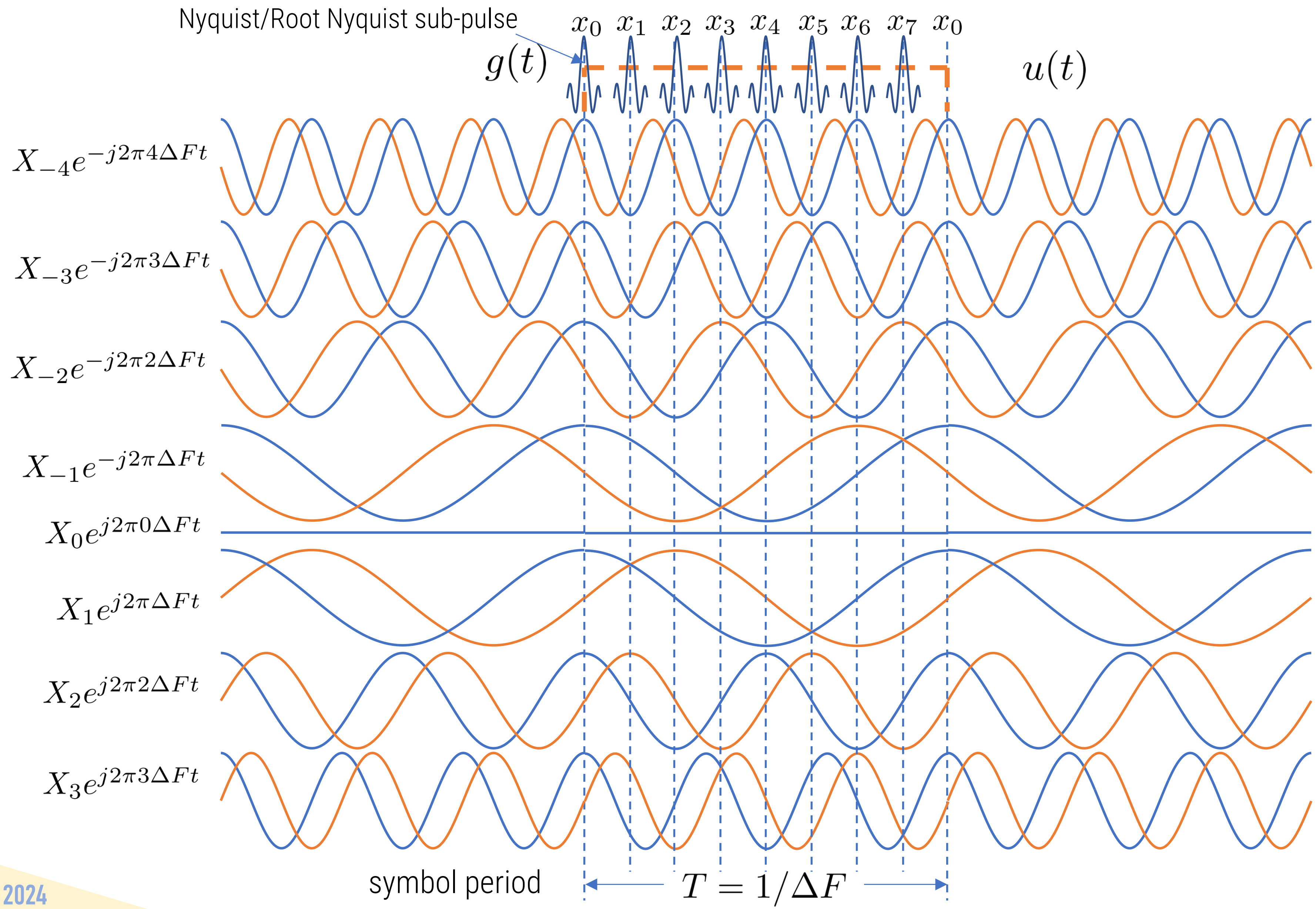
- In practical system, DD domain is an on-grid DD domain associated with the delay and Doppler resolutions.
- Due to the presence of frequency resolution, a DD domain modulation is naturally an MC modulation.

$$x(t) = \sum_{m=0}^{M-1} \sum_{n=-\frac{N}{2}}^{\frac{N}{2}-1} X[m,n] g(t - m\frac{1}{\mathbb{W}}) e^{j2\pi \frac{n}{\mathbb{T}}(t - m\frac{1}{\mathbb{W}})}$$

$$\Delta F = \frac{1}{\mathbb{T}} \qquad \Delta T = \frac{1}{\mathbb{W}} \qquad \Delta R = \frac{1}{\mathbb{W}\mathbb{T}} \ll 1$$

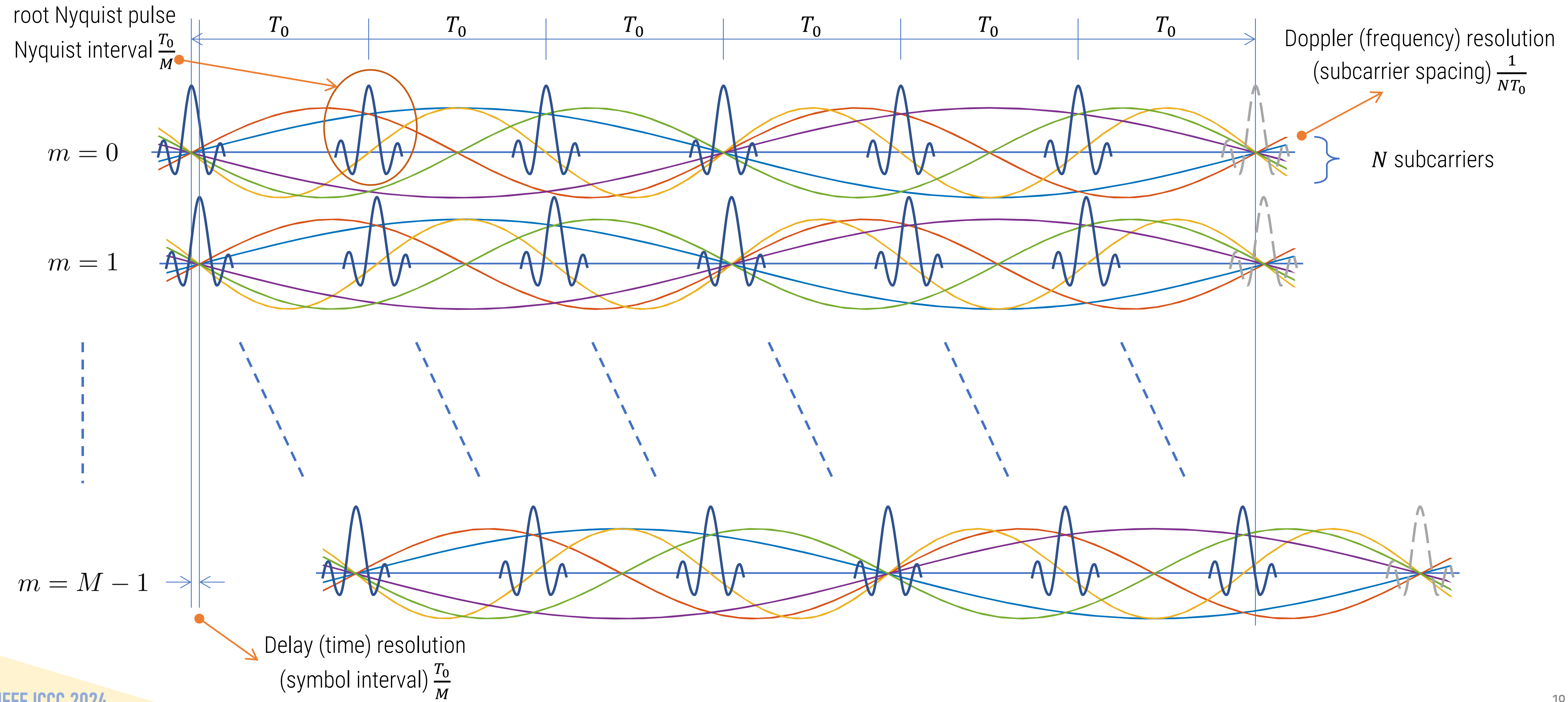
- We don't have DDLP (due to uncertainty principle)
- We don't have WH set orthogonal with respect to DD resolutions either (due to WH frame theory)
- No orthogonal pulse with long duration and wide bandwidth to design a DD domain MC modulation?

DDMC Modulation



- DD domain/plane orthogonal pulse **(DDOP)**
- A pulse-train can achieve the orthogonality among subcarriers
- Orthogonality among symbols can be achieved by employing Nyquist sub-pulse.
- WH subset based waveform design

Orthogonal Delay-Doppler Division Multiplexing (ODDM)



ODDM Waveform versus OTFS Waveform

Transmit pulse of TFMC/OFDM

$$s(t) = \sum_{\grave{n}=0}^{N-1} \sum_{\grave{m}=0}^{M-1} \mathcal{X}[\grave{m}, \grave{n}] g(t - \grave{n}T_0) e^{j2\pi \grave{m}\frac{1}{T_0}(t - \grave{n}T_0)}$$

$$\mathcal{X}[\grave{m}, \grave{n}] = \frac{1}{\sqrt{MN}} \sum_{m=0}^{M-1} \sum_{n=0}^{N-1} X[m, n] e^{j2\pi\left(\frac{\grave{n}n}{N} - \frac{\grave{m}m}{M}\right)}$$

OTFS

Ambiguity function/Orthogonality

$$A_{g,g}\left(nT_0, m\frac{1}{T_0}\right) = \delta(m)\delta(n)$$

- WH set based
- Global orthogonality

$$x(t) = \sum_{m=0}^{M-1} \sum_{n=-\frac{N}{2}}^{\frac{N}{2}-1} X[m, n] u\left(t - m\frac{T_0}{M}\right) e^{j2\pi \frac{n}{NT_0}\left(t - m\frac{T_0}{M}\right)}$$

$$u(t) = \sum_{\dot{n}=0}^{N-1} a(t - \dot{n}T_0)$$

DD domain orthogonal pulse

ODDM

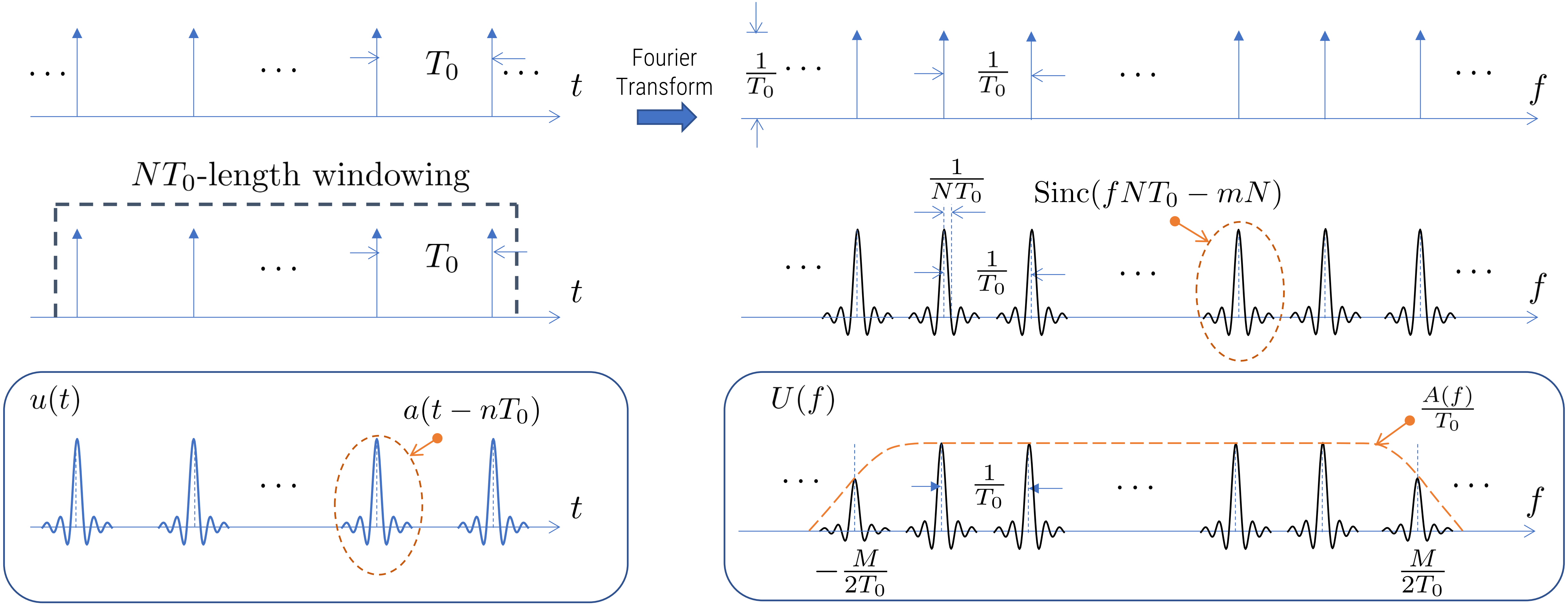
Ambiguity function/Orthogonality

$$A_{u,u}\left(m\frac{T_0}{M}, n\frac{1}{NT_0}\right) = \delta(m)\delta(n)$$

$$\forall |m| \leq M-1, |n| \leq N-1$$

- WH subset based
- Sufficient/Local orthogonality

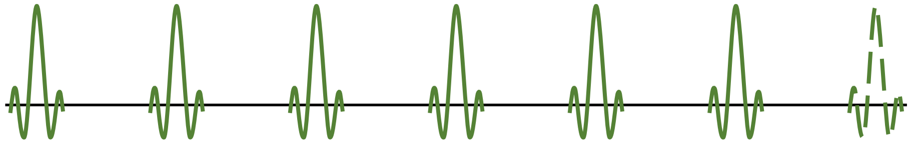
DD Domain Orthogonal Pulse (DDOP)



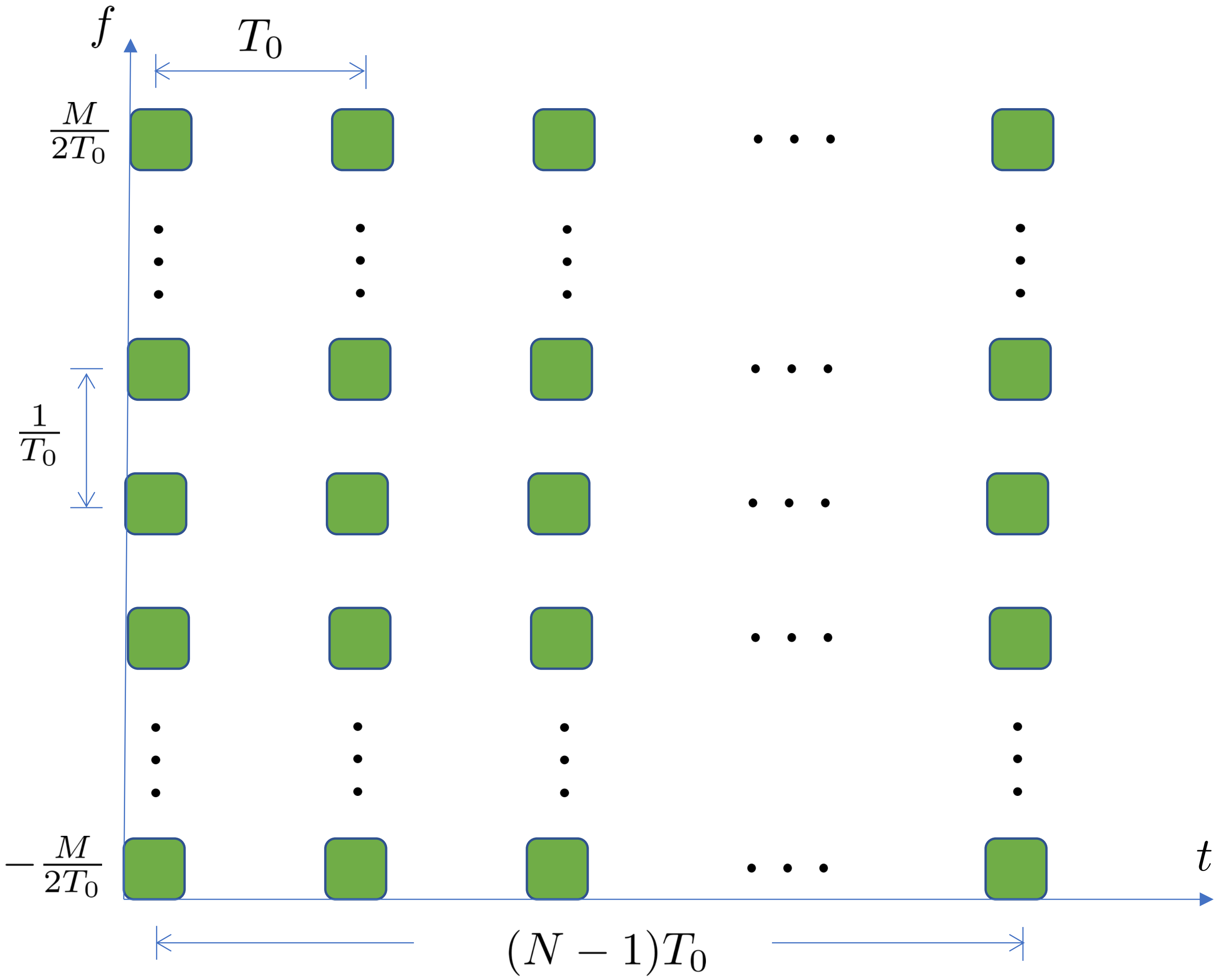
$$u(t) = \sum_{\dot{n}=0}^{N-1} a(t - \dot{n}T_0)$$

$$U(f) = \frac{e^{-j2\pi f\tilde{T}}}{T_0} A(f) \sum_{m=-\infty}^{\infty} e^{j2\pi \frac{m(N-1)}{2}} \mathrm{Sinc}(fNT_0 - mN)$$

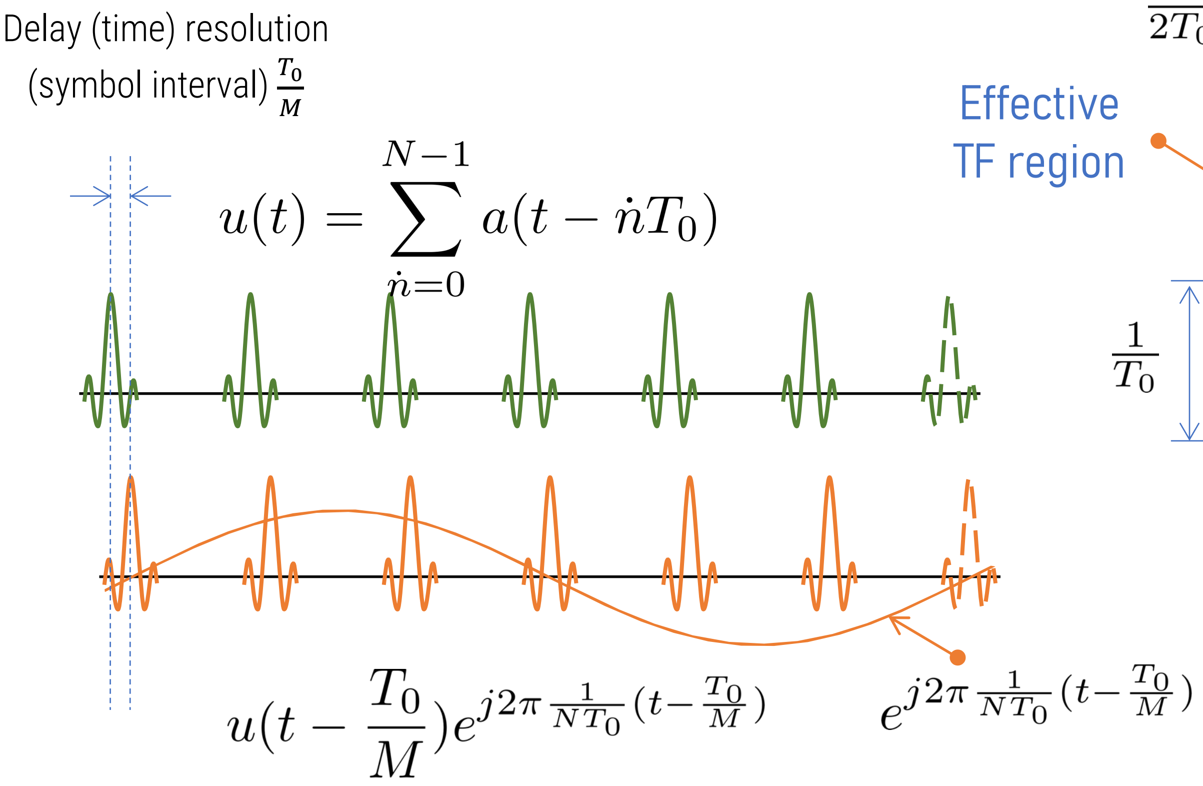
TF Signal Energy Localization of DDOP

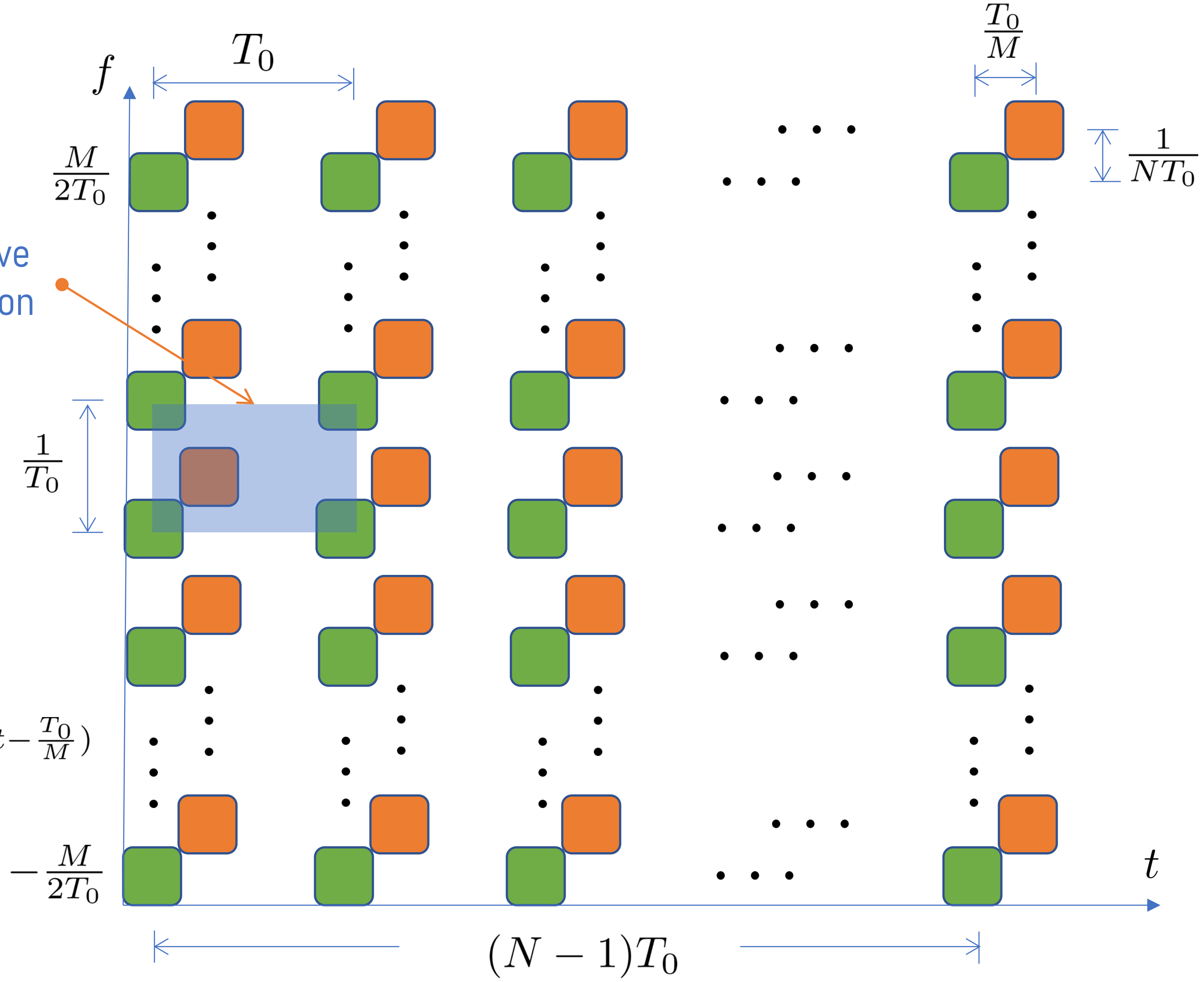


$$u(t) = \sum_{\dot{n}=0}^{N-1} a(t - \dot{n}T_0)$$

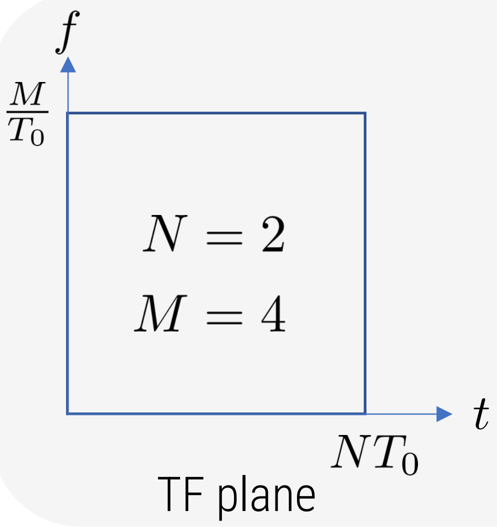


TF Signal Energy Localization of ODDM

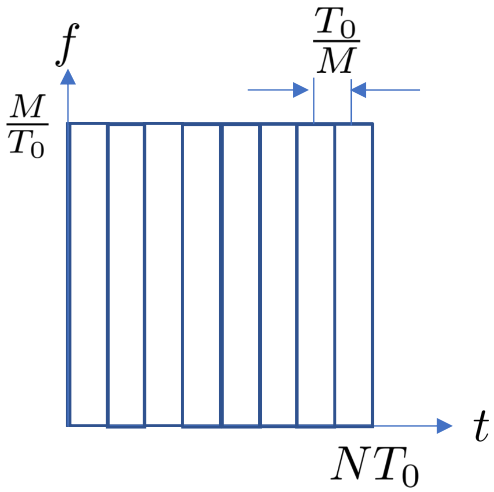




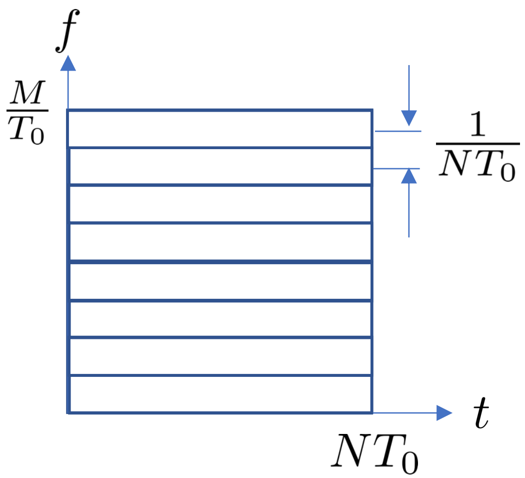
From the Viewpoint of DoF



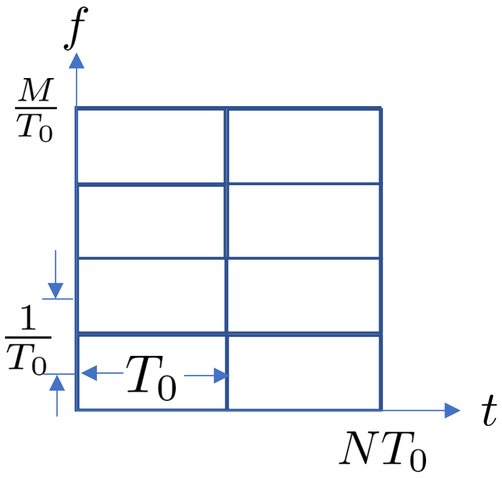
- Landau-Pollak Theorem
- DoF$\approx \frac{M}{T_0} \times NT_0 = MN$
- MN symbols



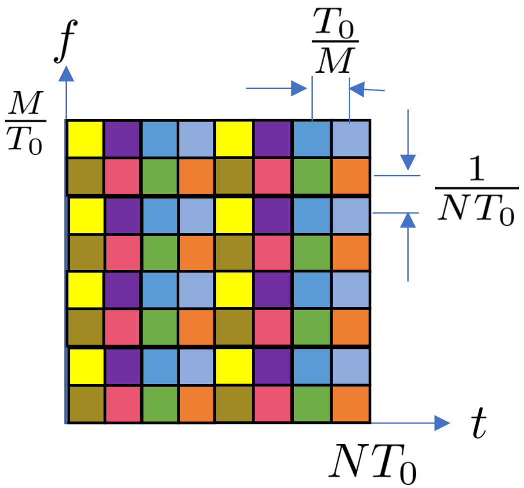
TDM(SC)
DoF: MN



FDM(OFDM)
DoF: MN

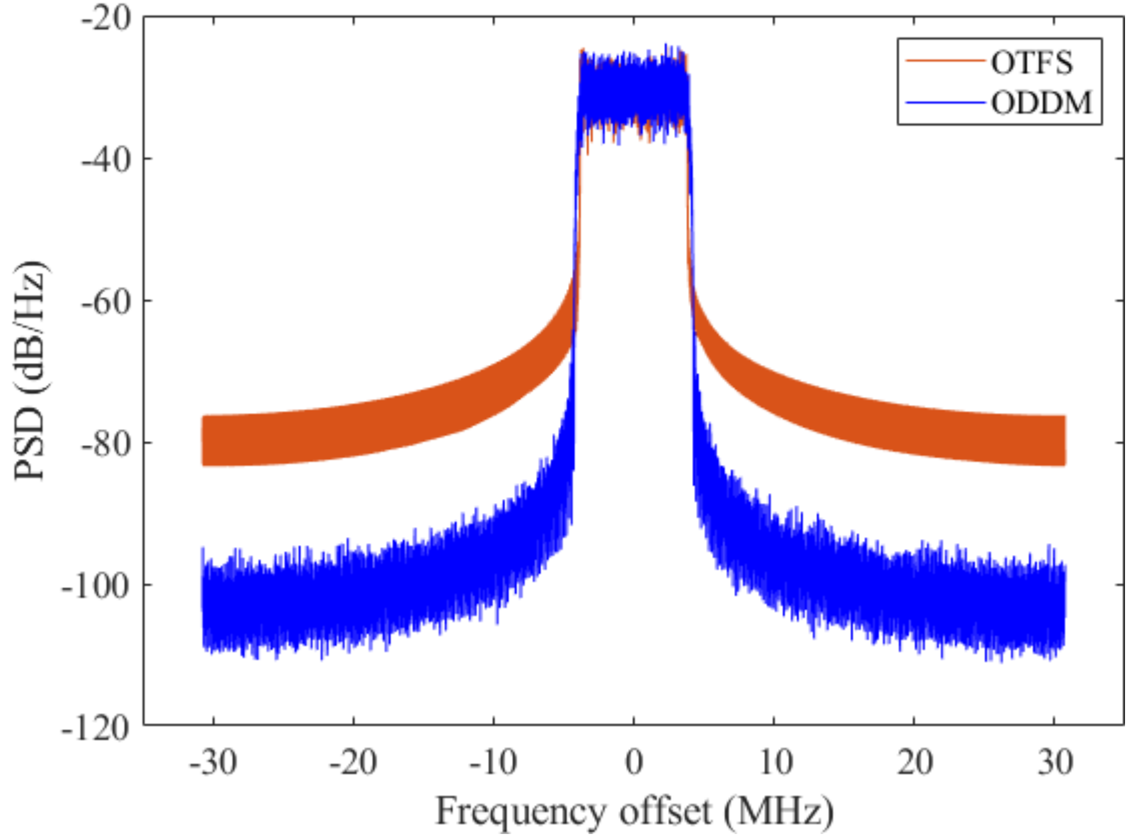


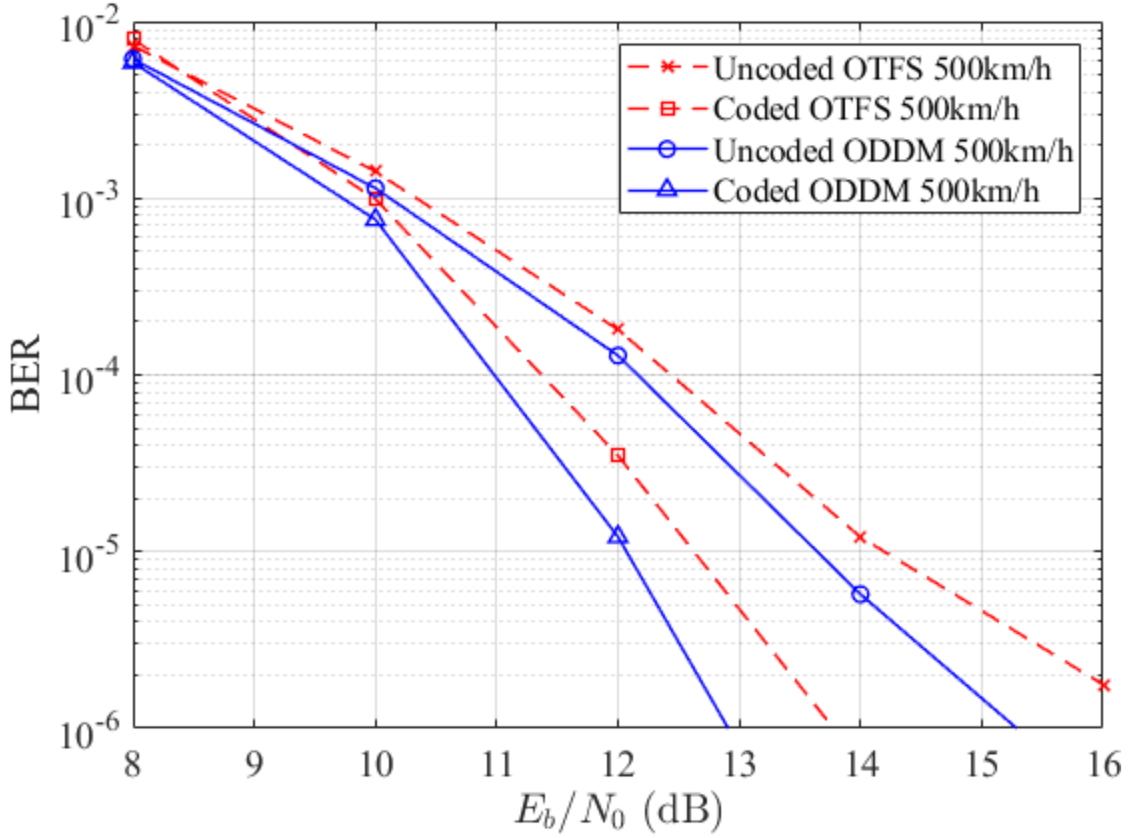
OFDM
DoF: MN



ODDM
DoF: MN

Waveform-Level Simulation Results





- $M = 512,\ N = 32,\ \frac{1}{T_0} = 15\text{kHz}, f_c = 5\text{GHz},$ EVA Channel
- $Q = 20,$ roll-off factor = 0.1, 4-QAM, MP Equalization

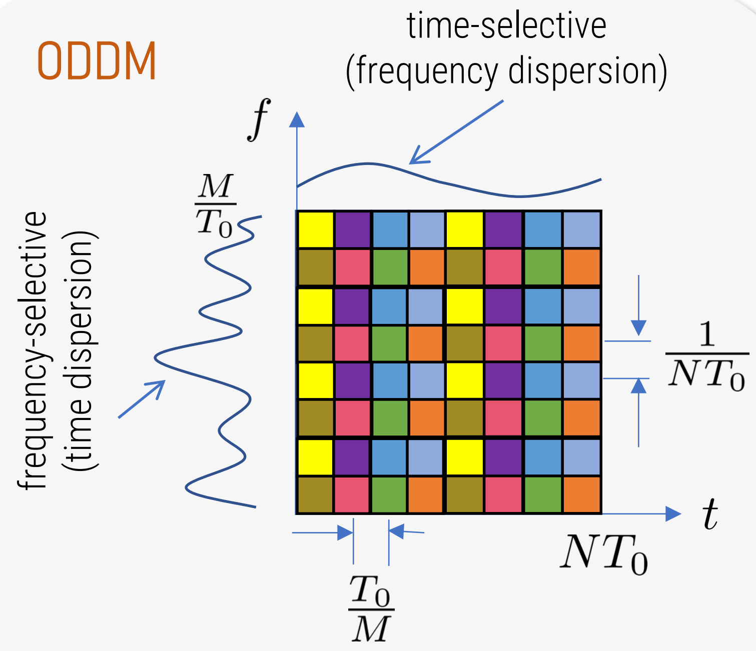
Conclusion

- A novel multi-carrier waveform
  - ✓ Embracing DD channel property
- DD domain/plane orthogonal pulse (DDOP)
  - Pulse train with long duration and wide bandwidth
  - Bypass the limitations imposed by the uncertainty principle and the WH frame theory
- Potential for future
  - ✓ Reliable Communication for High Mobility
  - ✓ Integrated Sensing and Communication (ISAC)
  - ✓ …
- Many open issues. More details will be posted on:

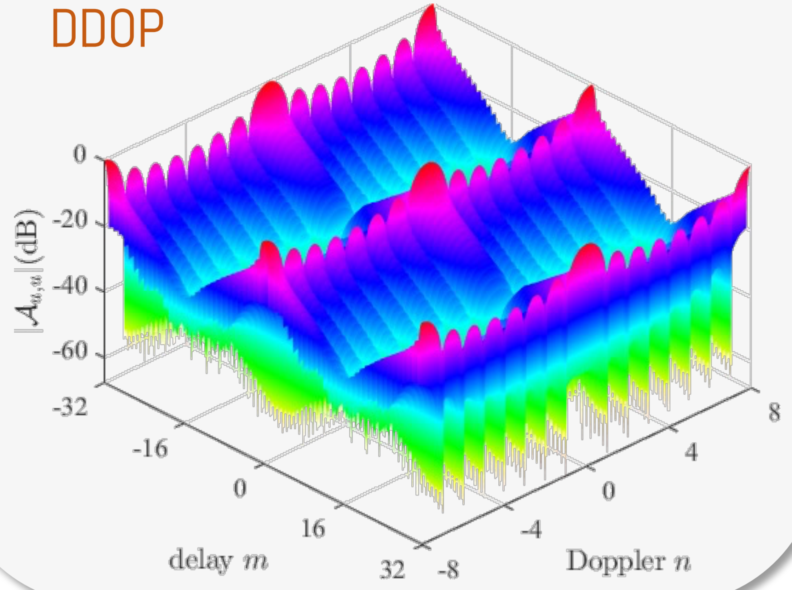
  https://www.omu.ac.jp/eng/ees-sic/oddm/

  https://oddm.io (coming soon)

ODDM



DDOP



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■ For more information, please Google or Bing "ODDM waveform"

Acknowledgment

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■ Questions or Comments : Please send to hai.lin@ieee.org

Thank you for your attention!